

Fig. 5—Curves of cut-off wavelengths  $\lambda_{c1}$  and  $\lambda_{c2}$  as a function of plate depth,  $d$  with both coordinates normalised to radius  $r$  of circular waveguide.

gives a value of  $\theta$  which, on substitution in (2), will determine the computed cut-off wavelength  $\lambda_c(N)$ .

In the above determination no account has been taken of the curvature of the electric field across the waveguide. Since the electric field is concentrated in the region in which it is least curved, this will lead to only a small error in determining the cut-off wavelength.

In order to determine this, error values of  $\lambda_c(N)$  are computed for a range of values of  $N$  and compared with the true value of  $\lambda_c$  which, for the  $H_{11}$  mode, is<sup>2</sup>

$$\lambda_c = 2\pi r / 1.841184 \dots \quad (8)$$

Defining the error  $\epsilon$  in determining  $\lambda_c$  by

$$\epsilon = \frac{\lambda_c - \lambda_c(N)}{\lambda_c(N)}, \dots \quad (9)$$

the variation of  $\epsilon$  with  $N$  follows the curve shown in Fig. 3. If computations are carried out with  $N$  greater than one thousand, the value of  $\epsilon$  remains steady at  $2.41 \times 10^{-2}$ , i.e., there is a known bias error in the computed results.

Once the system of rectangular strips has been established, it is a simple matter to modify or remove strips in order to simulate the unusual cross section (see Fig. 4).

In the case of metal plates perpendicular

to the electric field  $E_1$ , the heights of the first  $p$  strips are lowered to that of the  $p$ th strip [see Fig. 4(a)]. Proceeding as for the unperturbed circular waveguide, the cut-off wavelength  $\lambda_{c1}$  is determined from (7) and (2) and the predetermined bias error.

In the case of metal plates parallel to the electric field  $E_2$ , the last  $q$  strips are removed so that the short circuit appears at the end of the  $(N-q)$ th strip [see Fig. 4(b)]. The cut-off wavelength  $\lambda_{c2}$  is determined in the same way as for  $\lambda_{c1}$ .

The computation of  $\lambda_{c1}$  and  $\lambda_{c2}$  has been carried out on a programmed digital computer (IBM 7090). Values of  $N$  between 1400 and 2000 were used to obtain the curves shown in Fig. 5.

The method described in this communication may be applied to other waveguides of unusual cross section, provided the bias error is determined from a computation of  $\lambda_c$  for the closest well-known cross section.

#### ACKNOWLEDGMENT

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#### Circular Polarizers of Fixed Bandwidth

This communication describes the design and performance of a circular polarizer comprising two metal plates set at  $45^\circ$  to the electric field vector,  $E$  (Fig. 1) in a circular waveguide of radius  $r$  carrying the  $TE_{11}$  mode.

The bandwidth of this polarizer is fixed and practically independent of plate length.

At a free space wavelength  $\lambda$ , the differential phase shift constant,  $\beta_1 - \beta_2$ , may be expressed in terms of the cut-off wavelengths  $\lambda_{c1}$  and  $\lambda_{c2}$  for the component fields  $E_1$  and  $E_2$  (Fig. 1).

$$\beta_1 - \beta_2 = \frac{360}{\lambda} \left[ \sqrt{1 - \left(\frac{\lambda}{\lambda_{c1}}\right)^2} - \sqrt{1 - \left(\frac{\lambda}{\lambda_{c2}}\right)^2} \right] \quad (1)$$

degrees per unit length.

The cut-off wavelength  $\lambda_c$  of the pure  $TE_{11}$  mode is<sup>1</sup>

$$\lambda_c = 2\pi r / 1.841184 \dots \quad (2)$$

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<sup>1</sup> S. A. Schelkunoff, "Electromagnetic Waves," D. Van Nostrand Co., Inc., New York, N. Y., 1st ed., ch. 8, 1943.

<sup>2</sup> S. A. Schelkunoff, "Electromagnetic Waves," D. Van Nostrand Co., Inc., New York, N. Y., 1st ed., ch. 8, 1943.

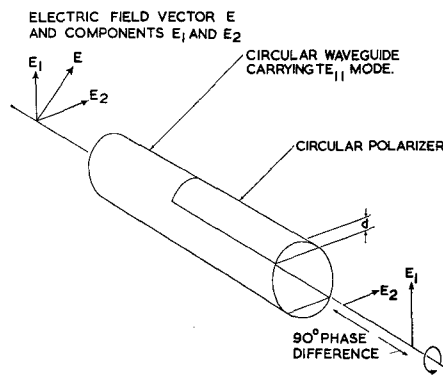
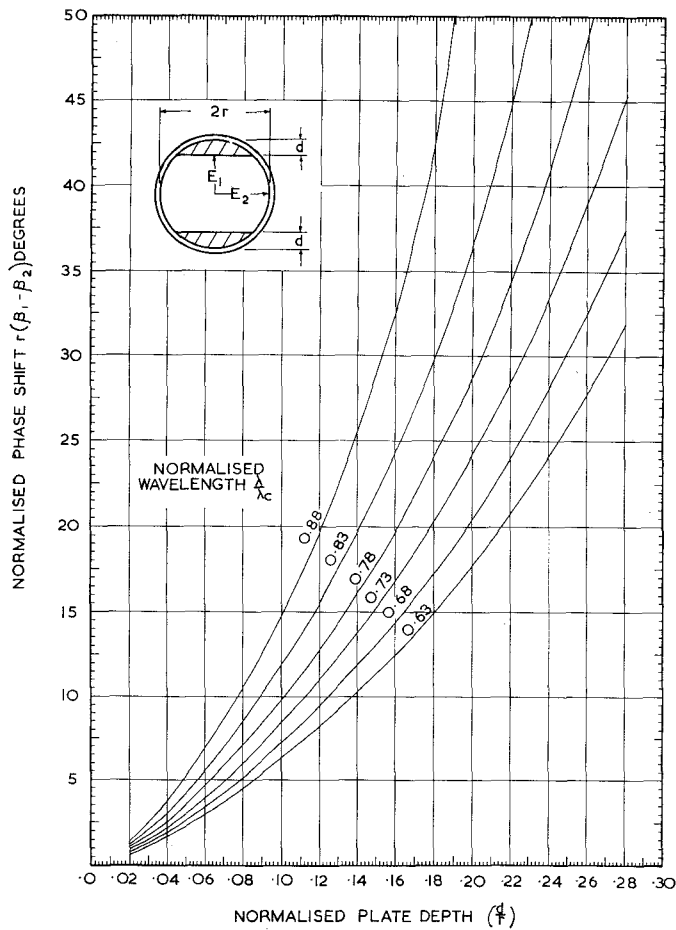
Fig. 1—Circular polarization of the  $TE_{11}$  mode.

Fig. 2—Design curves for circular polarizers.

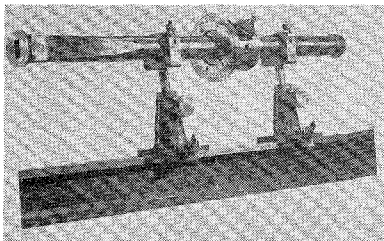
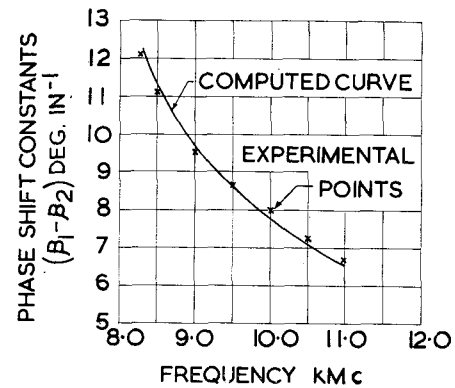
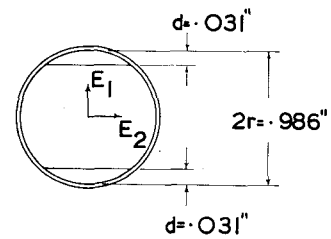
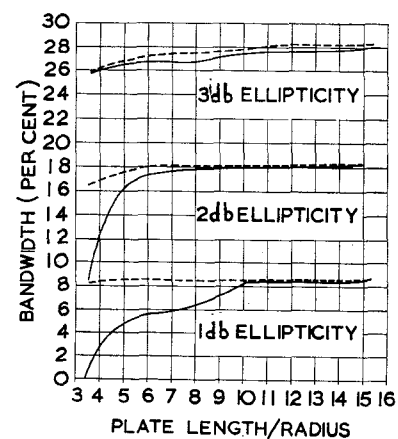
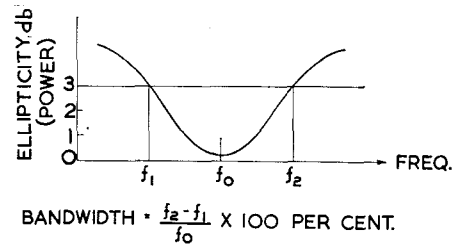


Fig. 3—An elliptically polarized field analyzer.

## CROSS-SECTION OF CIRCULAR POLARIZER

Fig. 4—Computed and experimental phase shift constants  $(\beta_1 - \beta_2)$  for 0.031 inch plates in 0.986 inch diameter circular waveguide.

LEGEND      --- MATCHED PLATES  
                   — UNMATCHED PLATES

Fig. 5—Bandwidth of circular polarizers as a function of plate length.

and the recommended<sup>2</sup> operating wavelength range is

$$\lambda = F\lambda_0 \quad \text{where} \quad 0.635 < F < 0.870.$$

The generalized expression for differential phase shift may be written

$$r(\beta_1 - \beta_2) = \frac{C_2}{F} \left[ \sqrt{1 - \left( \frac{C_1 F}{\lambda_{c1}/r} \right)^2} - \sqrt{1 - \left( \frac{C_1 F}{\lambda_{c2}/r} \right)^2} \right] \text{degrees} \quad (3)$$

where

$$C_1 = 2\pi/1.841184$$

$$C_2 = 360/C_1$$

and  $\lambda_{c1}/r$ ,  $\lambda_{c2}/r$  are known for values of  $d/r$  where  $d$  is the penetration depth of each metal plate

Curves (Fig. 2) of  $r(\beta_1 - \beta_2)$  vs  $d/r$  have been computed for values of  $F$  between 0.63 and 0.88. They may be used for the design of circular polarizers in any circular waveguide operated at a wavelength within the recommended range.

A circular polarizer was constructed using 0.031 inch plates in 0.986 inch diameter circular waveguide. An elliptically polarized field analyzer (Fig. 3) was used to measure the magnitude and orientation of the polarization ellipses obtained. These quantities suffice to determine<sup>3,4</sup> the actual phase shift introduced by the plate at each operating frequency.

<sup>2</sup> "Military Standard Specification MIL-W-23068," Armed Service Technical Information Agency, Arlington, Va., Rept. No. MIL-W-23068; October, 1961.

<sup>3</sup> "Very High Frequency Techniques," Radio Research Lab. Staff, Harvard University, Cambridge, Mass., published by McGraw-Hill Book Co., Inc., 1st ed., vol. 1, ch. 6, 1947.

<sup>4</sup> "Reference Data for Radio Engineers," International Telephone and Telegraph Corp., published by Stratford Press Inc., New York, N. Y., 4th ed., ch. 23, pp. 666-670; 1961.

A comparison between computed and experimentally determined phase shift constants is shown in Fig. 4. There is agreement to within 0.2° per inch, this being the maximum deviation of any experimental point about the computed curve.

The percentage bandwidths obtainable with this type of polarizer have been computed for plates from three to fifteen radii in length (Fig. 5). The bandwidth to 3 db ellipticity is approximately constant ( $27 \pm 1$  per cent) for plates greater than four radii in length. Matching of the plates by some form of stepping is normally unnecessary.

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terdigital filters of prescribed bandwidth and number of cavities  $N$ . The low-pass prototype elements are found from tables.<sup>2,3</sup> The widths and spacings of the stripline resonators are determined using Getsinger's<sup>4</sup> charts of capacitance for coupled rectangular bars between parallel plates.

Experience has shown that to design one band-pass filter using the above references may take up to two hours especially if the operator is unfamiliar with the procedure. The author has used an IBM 7090 computer to carry out all the operations necessary for the design of an interdigital band-pass filter. The program<sup>5</sup> consists of about 200 cards and requires 2 seconds execution time per filter.

Figs. 2 to 9 (pages 560-567) give computed widths  $\Pi_K$  and spacings  $S_{K,K+1}$  of interdigital stripline arrays of  $N$  cavities. The curves may be used to design maximally flat and Chebyshev type filters having bandwidths up to ten per cent and values of  $N$  between three and eight. Filter parameters are given in inches for the commonly used 0.0625 inch brass strip between ground planes spaced at 0.3125 inch (i.e.,  $t/b = 0.2$ ). Designers may use the curves for different ground plane spacings,  $b$  provided that  $t/b = 0.2$ , and the values of  $\Pi_K$  and  $S_{K,K+1}$  are multiplied by  $b/0.3125$ .

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### Design Curves for Interdigital Band-Pass Filters

Band-pass filters may be constructed using interdigital arrays of quarter-wave stripline resonators as shown in Fig. 1. Interdigital filters are attractive in that they are

- 1) compact
- 2) easily fabricated without tight tolerances
- 3) free of spurious responses at twice the pass-band frequency,  $f_0$  (second pass-band is at  $3f_0$ ).

Matthaei<sup>1</sup> gives design equations for in-

Manuscript received June 1, 1964.

<sup>1</sup> G. L. Matthaei, "Interdigital band-pass filters," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-10, pp. 479-491; November, 1962.

<sup>2</sup> "The Microwave Engineers' Handbook and Buyers' Guide," p. 95, 1964.

<sup>3</sup> L. Weinberg, "Additional tables for design of optimum ladder networks," J. Franklin Inst., vol. 246, pp. 7-23, 127-138, July and August, 1957.

<sup>4</sup> W. J. Getsinger, "Coupled rectangular bars between parallel plates," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-10, pp. 65-72; January, 1962.

<sup>5</sup> J. R. Pyle, "IBM Programme I.G.4.10.26" Weapons Research Establishment; January, 1964.

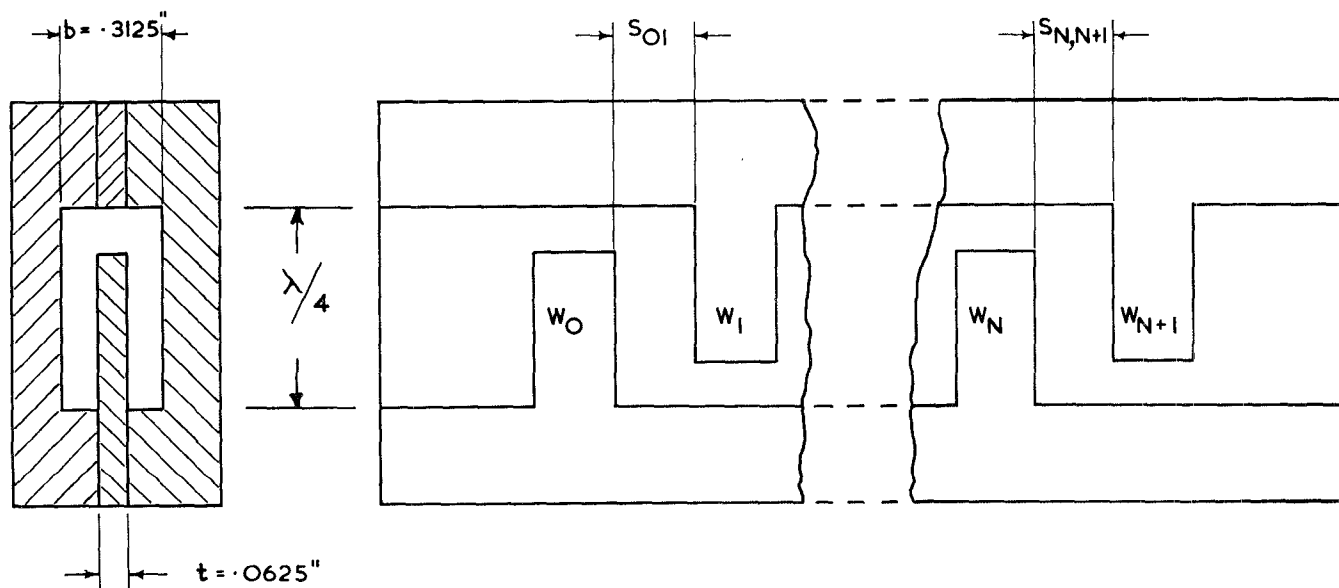


Fig. 1—Construction of an interdigital stripline band-pass filter array of  $N$  cavities.